A Study of Symmetry Breaking Predicates and Model Counting

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1. University of Texas at Austin, Austin, TX, USA
2. Google Inc., Sunnyvale, CA, USA
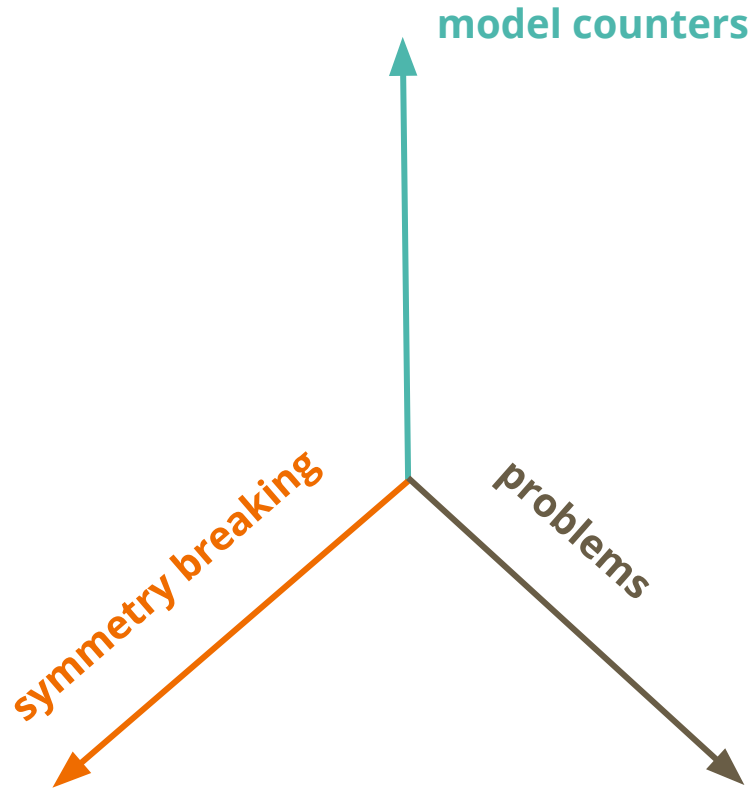3. National University of Singapore, Singapore

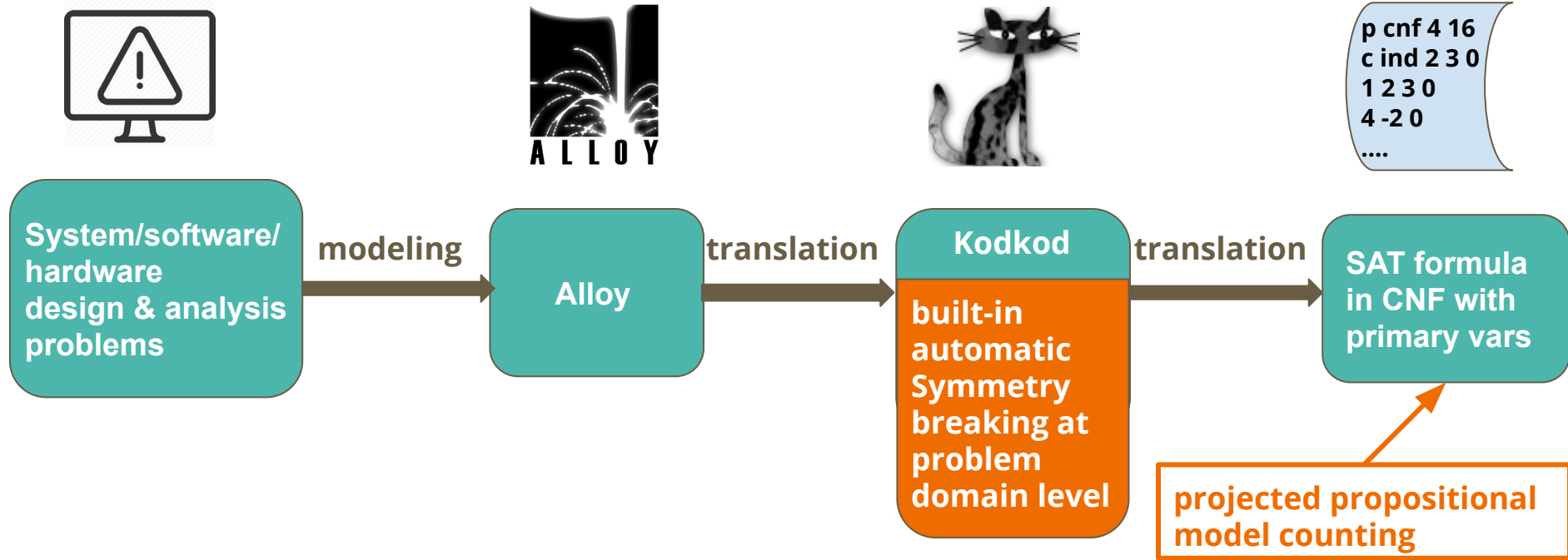# What we want to study?

What is the impact of symmetry breaking on model counting?

How we did the study?

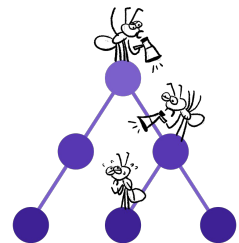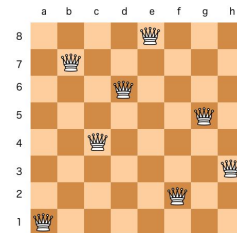# Problems - Alloy & Kodkod for CNF generator

Problems - base benchmarks

- **Alloy:** all Alloy models in Alloy standard distribution; **47** base problems.

- **Kodkod:** all Kodkod programs in Kodkod standard distribution; **13** base problems.

- **n-Queens:** 1) k queens are placed on a k × k board (1 ≤ k ≤ 12); 2) 3 queens are placed on a k × k board (1 ≤ k ≤ 12); **24** base problems.

- **Complex data structures:** (1) singly-linked lists; (2) sorted lists; (3) doubly-linked lists; (4) binary trees; (5) binary search trees; and (6) red-black trees; **24** base problems;

Symmetry Breaking

- **Static symmetry breaking => symmetry breaking predicates (SBPs).**
- **CNF level SBPs:**

  use the state-of-the-art tool called **BreakID**;

- **problem domain level SBPs:**
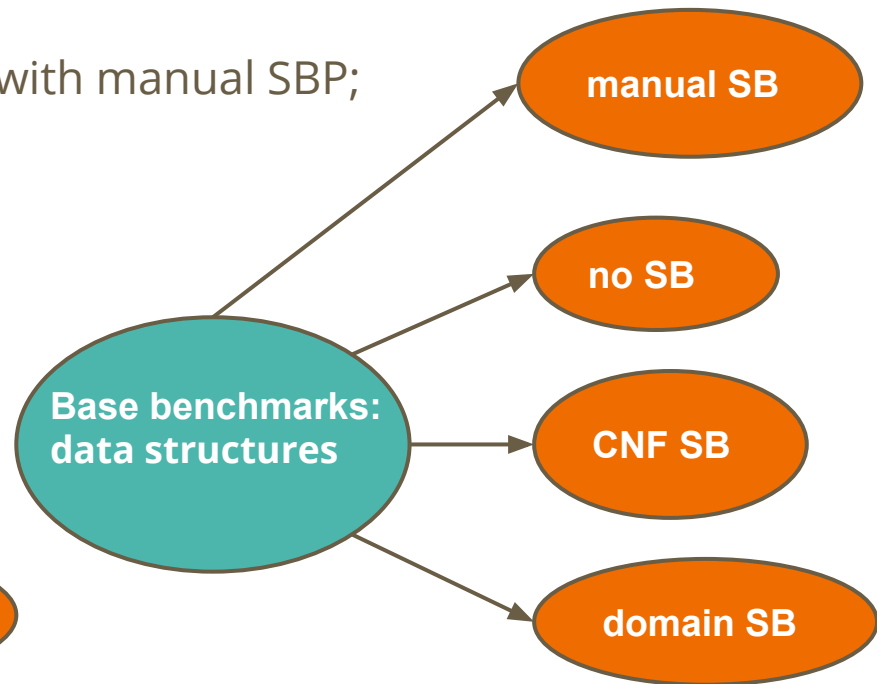
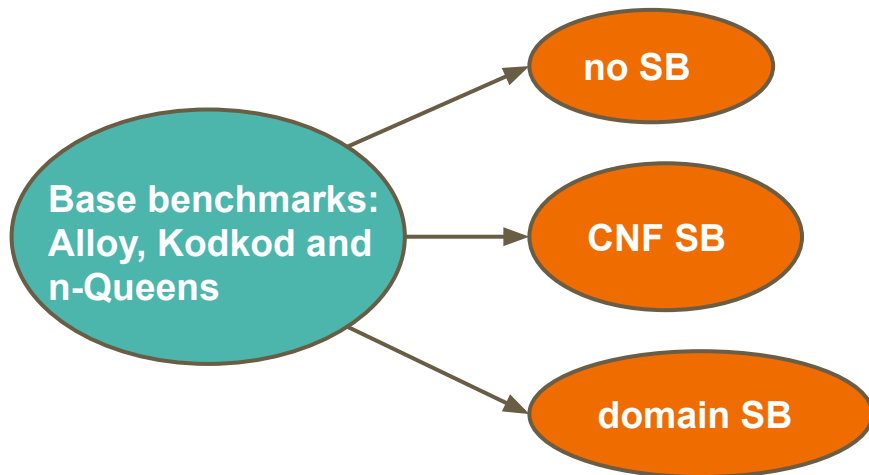 use **Kodkod automatic SB machanism**.

- **manually added SBPs:**

  write by human in **Alloy**.

Symmetry Breaking - benchmarks for study

For each base benchmark *b, we create*:

- *b* with no symmetry breaking;
- *b* with CNF level SBP;
- *b* with problem domain SBP;
- For **data structure benchmark**: *b* with manual SBP;

Problems & Symmetry breaking - benchmark characteristics.

source	#prim.	no-sb		cnf-sb		dom-sb		man-sb	
		#var.	#clause	#var.	#clause	#var.	#clause	#var.	#clause
Alloy: min	46	384	620	522	1037	384	620	-	-
Alloy: max	2048	93764	291349	93764	289725	93764	291349	-	-
Kodkod: min	48	631	188	932	628	990	188	-	-
Kodkod: max	8188	388755	764957	397566	834629	453358	877429	-	-
n-Queens: min	1024	3762	7163	3762	7163	3762	7163	-	-
n-Queens: max	12288	200074	532527	201064	523947	269141	704396	-	-
Data Str.: min	43	992	3039	1091	3337	1209	3401	1006	3155
Data Str.: max	510	18694	48290	19045	45562	19808	50212	18993	50696

Model Counters

- **ApproxMC:**

  one of the state-of-the-art projected apprximate model counters.

- **ProjMC:**

  one of the state-of-the-art projected exact model counters.

  ApproxMC and ProjMC embody very different algorithms for model counting and provide us a diverse set of tools for the study.

Experimental Metrics

- **Time:**

  actual wall-clock times; 5000 seconds Timeout;

- **Model count:**

  1) actual count;

  2) count ratio:

  the ratio of the count under no symmetry breaking setting to the count under one symmetry breaking setting;
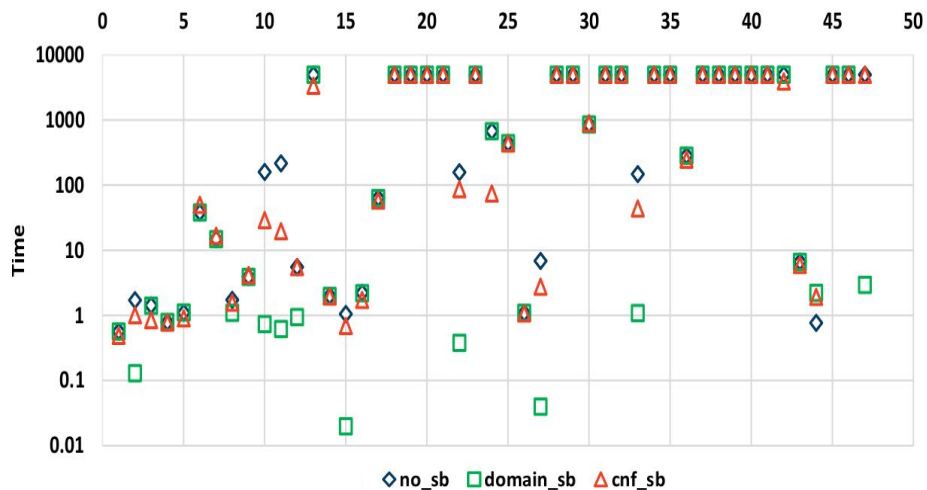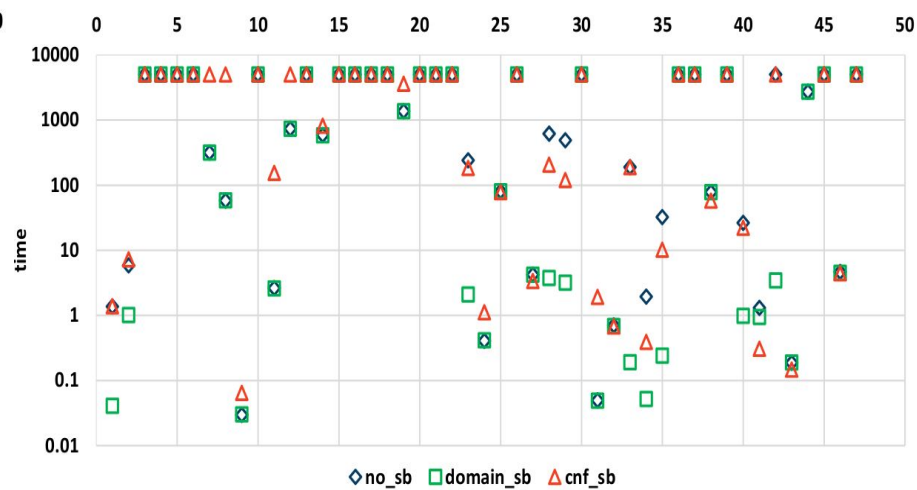
  the impact of SBP in solution space pruning
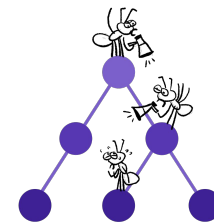
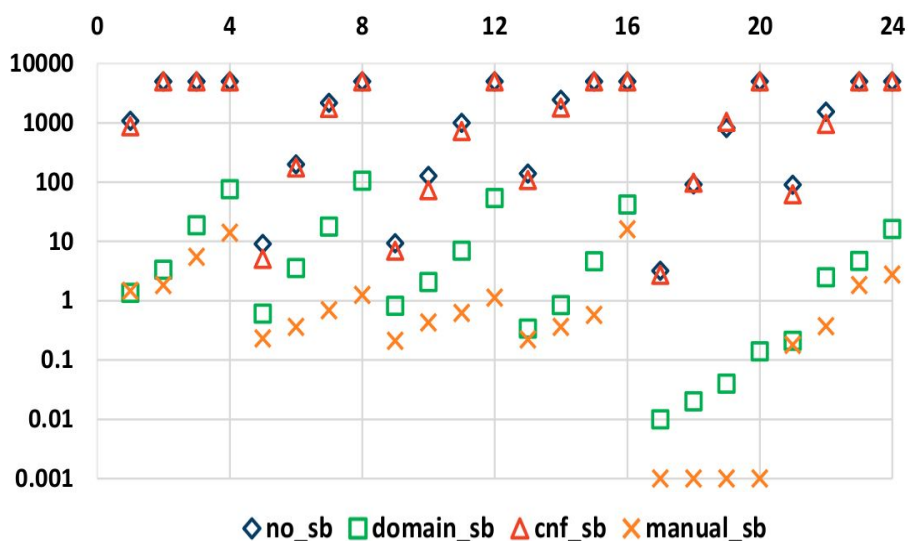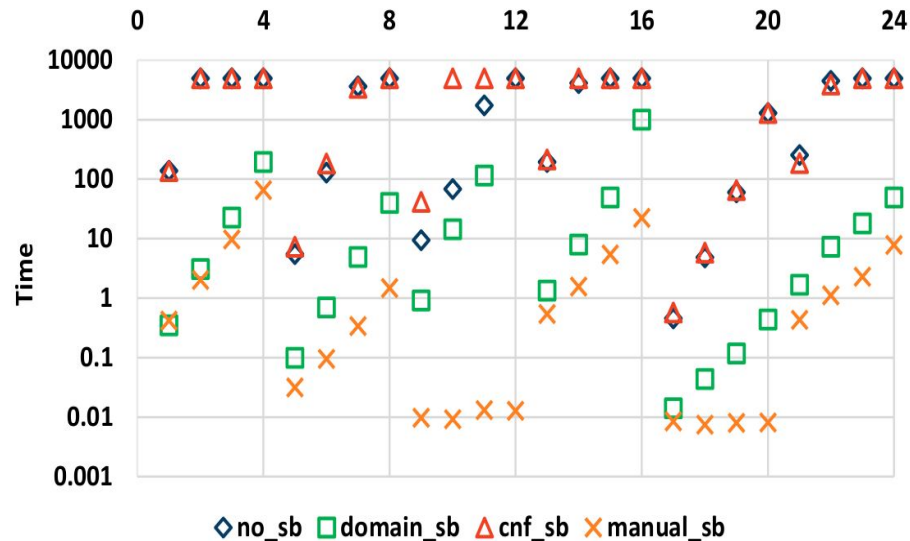# Results- Time in Alloy Problems

ApproxMC

ProjMC

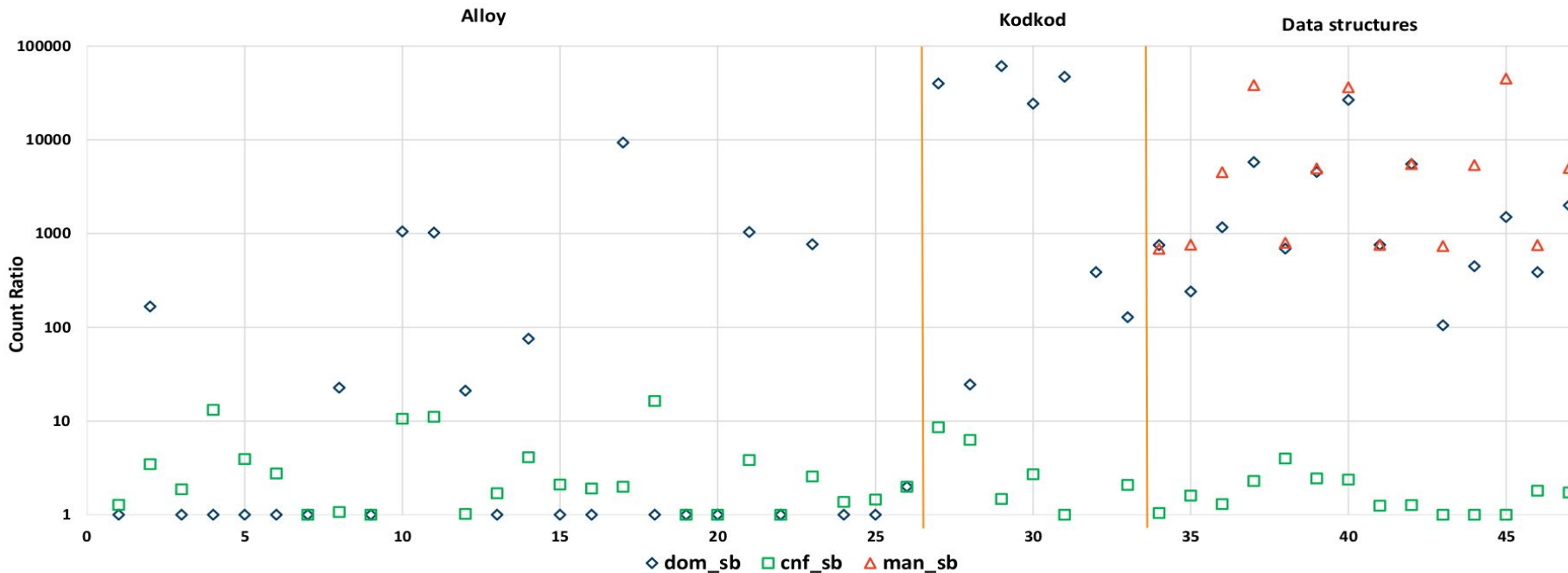# Results - Time in Data Structure Problems
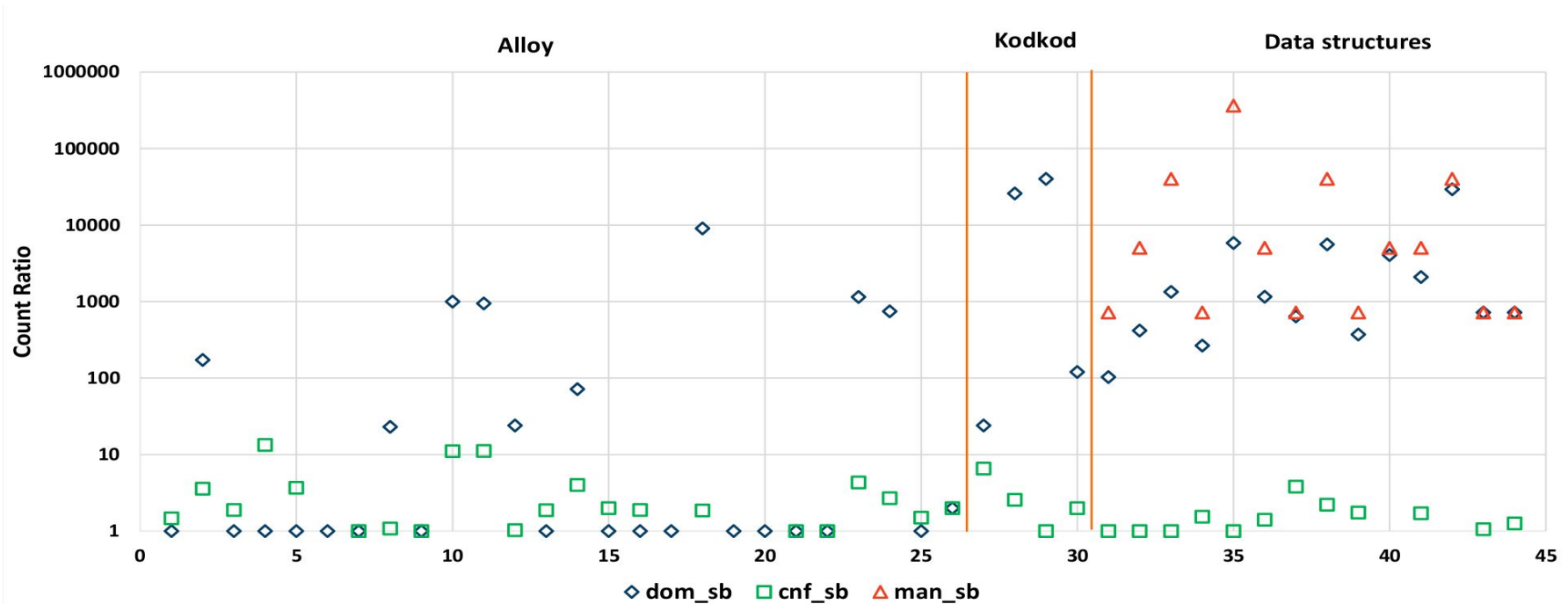
ApproxMC

ProjMC

Results - count ratios for ApproxMC under different SB settings

count ratio: the ratio of the count under no symmetry breaking setting to the count under one symmetry breaking setting;

Results - count ratios for ProjMC under different SB settings

Key Findings

- **Addition of symmetry breaking predicates can significantly reduce the time taken by model counters;**

- **Problem domain level symmetry breaking is more effective than CNF level symmetry breaking;**

Thank you for listening!

Any questions?